

## Can collective searches profit from Lévy walk strategies?

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2009 J. Phys. A: Math. Theor. 42 434017

(<http://iopscience.iop.org/1751-8121/42/43/434017>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 171.66.16.156

The article was downloaded on 03/06/2010 at 08:18

Please note that [terms and conditions apply](#).

# Can collective searches profit from Lévy walk strategies?

M C Santos<sup>1</sup>, E P Raposo<sup>2</sup>, G M Viswanathan<sup>3,4</sup> and M G E da Luz<sup>1</sup>

<sup>1</sup> Departamento de Física, Universidade Federal do Paraná, Curitiba-PR, 81531-990, Brazil

<sup>2</sup> Laboratório de Física Teórica e Computacional, Departamento de Física, Universidade Federal de Pernambuco, Recife-PE, 50670-901, Brazil

<sup>3</sup> Instituto de Física, Universidade Federal de Alagoas, Maceió-AL, 57072-970, Brazil

<sup>4</sup> Consortium of the Americas for Interdisciplinary Science, University of New Mexico, 800 Yale Blvd NE, Albuquerque, NM 87131, USA

E-mail: [luz@fisica.ufpr.br](mailto:luz@fisica.ufpr.br)

Received 16 July 2009, in final form 13 August 2009

Published 13 October 2009

Online at [stacks.iop.org/JPhysA/42/434017](http://stacks.iop.org/JPhysA/42/434017)

## Abstract

We address the problem of collective searching in which a group of walkers, guided by a leader, looks for randomly located target sites. In such a process, the necessity to maintain the group aggregated imposes a constraint in the foraging dynamics. We discuss four different models for the system collective behavior, with the leader and followers performing Gaussian as well as truncated Lévy walks. In environments with low density of targets we show that Lévy foraging is advantageous for the whole group, when compared with Gaussian strategy. Furthermore, certain extra rules must be incorporated in the individuals' dynamics, so that a compromise between the trend to keep the group together and the global efficiency of search is met. The exact character of these rules depends on specific details of the foraging process, such as regeneration of target sites and energy costs.

PACS numbers: 05.50.Fb, 05.40.-a

## 1. Introduction

It is widely known that random searches are relevant in many diverse contexts, such as anomalous diffusion, light scattering in inhomogeneous materials, ecology, genetics and control theory, just to name a few [1, 2]. An important related question regards search optimization, i.e. to determine specific strategies maximizing the ratio between the number of encounters and an appropriate cost function [3–6]. Search processes frequently involve constraints which generally limit their efficiency, e.g. dissipative losses and necessity of keeping collective behavior for a long term. In particular, the study field of collective searching [7] has attracted increasing attention in the past few years. In this case, a group, which may

be constituted by a leader and followers, looks collectively for randomly distributed targets, so that the intake is shared by all.

Establishing a group can be advantageous for many reasons, such as an exchange of information and an overall increase in the collective ability to find targets [8]. Interestingly, the gains associated with cooperative behavior are not restricted to biological organisms, but can emerge as well in other contexts, as robots and crawlers performing team search on networks [9, 10]. In addition, the flexibility of self-organized groups [11] can also be very helpful when the searching landscape is in constant change [12]. Actually, for critical situations, e.g. the edge of starvation [13], cooperation might become a fundamental ingredient to avoid extinction.

In spite of the above advantages, collective search is not the only (or even the most important) purpose of a group. Its formation can bear on extremely complex mechanisms [7, 14, 15], which are not yet fully understood. In the realm of animal (and even human) behavior, it is easy to think about many different driving forces for grouping: parental bounding, duty sharing, mutual defense, transmission of past experiences (knowledge), etc. Certainly, these many factors governing the group dynamics influence the strategies used for random search. Indeed, a nice glance at the many aspects of the problem can be found in a series of interesting works that study, both experimentally and theoretically, the foraging behavior of groups of spider monkeys in the Yucatan peninsula in Mexico [16–19]. The individuals are found to search following Lévy walk strategies, but the distributions of step lengths present distinct exponents for males and females, a result which probably reflects the different tasks the members of the group must accomplish within the community. As a consequence, it seems clear that, in general, the searching behavior of a group is not just a straightforward extension of the dynamics of a single element [20]. Furthermore, certain simplifying assumptions used to describe collective searching can even lead to unrealistic conclusions, as exemplified in [21]. On the other hand, including all the complex interactions among individuals in a group may yield too complicated a model.

In this work, we address a somewhat but nevertheless still relevant question for the understanding of this very complex problem. It is known that, under certain conditions, Lévy search strategies result in optimal efficiency for a sole searcher [1, 3, 5, 13], behavior empirically verified for a large number of different species [1, 22]. Here, we are interested in studying how, in a collective search, a group can profit from Lévy strategies, although still keeping its structure.

We should note that there are many instances in which a collective search takes place with all elements having exactly the same role within the group [10, 20]. However, the development of a hierarchical structure is also common, in which the group defines a leader for the search [14]. Indeed, a large set of empirical evidence for various animal species [23] supports the idea that single individuals can make decisions for the whole group (which sometimes is called ‘despotic’ behavior). Even more interestingly, experiments carried out with a particular species of primate during foraging [24] have shown the emergence of a leader–followers relationship. Theoretical studies [25] have also revealed that in certain situations concrete advantages (e.g. to reach a desired common speed or to get to a certain final destination) are more easily achieved with this kind of searching. Thus, our analysis will be aimed at such leader–followers dynamics.

The work is organized as follows: in section 2 we present general considerations on the group dynamics, and define some relevant quantities for the subsequent analysis. We also detail each of the four collective models considered, comprising both Gaussian and truncated Lévy behavior of the leader and followers. Section 3 is devoted to the analysis of collective search efficiency using an incremental version of the truncated Lévy model, which is shown

to provide compatibility between efficient search and maintenance of the group character. Conclusions are left to section 4.

## 2. Models of collective dynamics

We define four models of collective dynamics, generally based on ‘follow-the-leader’-type strategies.

A group is here defined as composed of a leader element and a number of followers, which are reciprocally related through a set of dynamic rules of movement in two dimensions. We keep the present models as simple as possible, although still capturing the essence of the collective behavior. Specifically, the stochastic character of the movement of the leader and followers is implicit in the probability density functions (pdfs) for step lengths,  $p_L(\ell)$  and  $p_F(\ell)$ , and direction angles (with respect, e.g., to the  $x$ -axis),  $\omega_L(\theta)$  and  $\omega_F(\theta)$ , where subindices  $L$  and  $F$  concern the leader and followers, respectively.

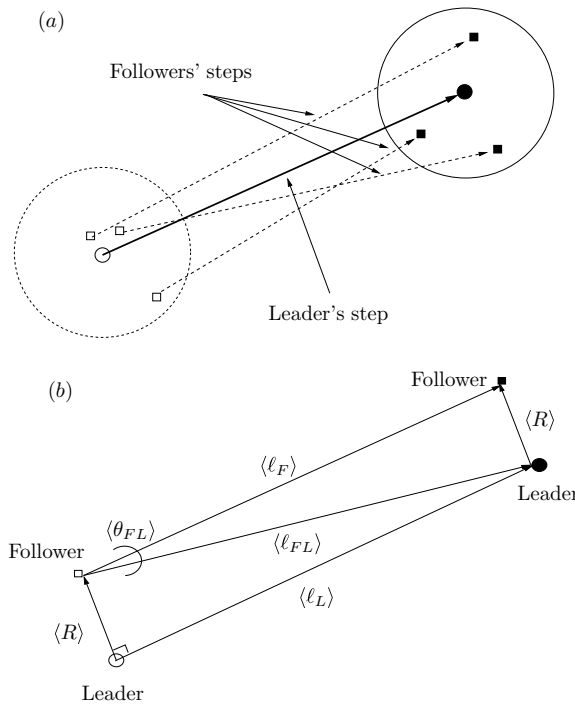
We generally assume that the interactions among the elements of the group are effectively intermediated by the leader, so that the followers’ paths do not depend on each others, even though they fluctuate around the trajectory described by the leader. In this sense, a direct stimulus–response correlation is induced between the leader and followers, which determines the values of the mean radius of the group and coefficient of separation between the group and the leader, respectively denoted by  $R_j$  and  $C_j$  (see below). Under such a framework, the collective character of a group is sustained if such quantities do not increase considerably along the walk trajectory. We also mention that in this work we are not considering the realistic possibility of group fragmentation, which occurs, for instance, when  $R_j$  and  $C_j$  become so large that the followers lose the capacity to accompany the leader, since it is too far away. Nevertheless, we should stress that, although in the present models the followers can always detect the leader’s position no matter its distance, a specific dynamics leading to strong fluctuations in these quantities (e.g. if  $R_j$  and  $C_j$  become larger than the actual followers’ skill to perceive the leader’s displacement) effectively indicate that the collective character of the group could be lost with such a strategy.

We first discuss the behavior of the leader. We choose a uniformly random  $\omega_L(\theta)$  pdf and a  $p_L(\ell)$  function either Gaussian (model A) or truncated Lévy (models B, C and D; see below). In the latter, one has that, for any step,

$$p_L(\ell) \sim \frac{1}{\ell_L^\mu}, \quad \ell_0 \leq \ell \leq \ell_{\max,L}, \quad (1)$$

and  $p_L(\ell) = 0$  otherwise, with the lower cutoff  $\ell_0$  representing the minimum step length and the maximum step length denoted by  $\ell_{\max,L}$ . (For reasons that become clear in section 3, here we set  $\ell_0 = r_v$ , where  $r_v$  is the so-called radius of vision [3].) Lévy walks and flights are characterized by the existence of rare but extremely large steps, alternating between sequences of many short-range jumps [2]. Indeed, it is clear from equation (1) that the smaller the Lévy index  $\mu_L$ , the larger the probability of long jumps becomes, with the ballistic limit reached for  $\mu \rightarrow 1$  (values  $\mu \leq 1$  do not correspond to normalizable pdfs).

In the context of realistic searches, generally involving dissipative processes associated with the step length, truncation arises naturally due to the physical impossibility of infinitely large steps. As a consequence, the genuine anomalous (superdiffusive) dynamics, observed for  $1 < \mu_L \leq 3$  when  $\ell_{\max,L} \rightarrow \infty$  [2], becomes limited to some typical characteristic length scale related to  $\ell_{\max,L}$  [26]. Indeed, a diffusive dynamics governed by the central limit theorem emerges beyond this threshold, although it has been shown [27] that, in the truncated Lévy case, an astronomically large number of steps is needed before entering the Brownian regime.



**Figure 1.** (a) ‘Follow-the-leader’-type dynamics in two dimensions, showing the leader and three followers. As the former takes a step, the followers move according to the rules described in the text in order to keep grouped in a compact way around the leader. (b) Illustration of average quantities related to a ‘mean follower’ and the leader.

Thus, for  $1 < \mu_L \leq 3$  and our choice of parameters we shall actually be working in the effective superdiffusive regime [28] of truncated Lévy walks of the leader, so as to retain the most important properties of non-truncated Lévy processes to a considerable extent.

On the other hand, for  $\mu_L > 3$  a diffusive Brownian search takes place, independently of the value of  $\ell_{\max,L}$ . In this case, the statistical properties of the leader’s path are similar to those of a walk generated by a Gaussian choice for  $p_L(\ell)$  [29].

We now turn to the dynamics of the followers. For a group to remain essentially compact, and continuously subject to the trends of a leader after  $j$  steps, the followers should lie within a relatively small radius  $R_j$  around the leader as its path evolves (see our operational definition of  $R_j$  below). Otherwise, if followers start to diverge from the leader’s position, the collective character of the process is effectively lost.

There are many possible ways to implement dynamic rules of movement that lead to finite values of  $R_j$  (not necessarily constant with path evolution). Our choices are described as follows. First, suppose that after a certain number of steps the group is spatially distributed around the leader, which then takes its next (say  $j$ th) step to a new position by following the rules described above. Let  $\tilde{\theta}_{i,j}$  be the angle that should be taken by the follower  $i$  to reach the new position of the leader. In all our models (see figure 1(a)) the actual direction angle of the follower  $i$  is taken from a Gaussian  $\omega_F(\theta)$ -pdf centered about  $\tilde{\theta}_{i,j}$ , with the standard deviation  $\sigma_\theta$ . Moreover, we consider the following choices for the followers’ step length distribution  $p_F(\ell)$  (see details of the specific models below): Gaussian (models A and B), with mean

$\bar{\ell}_{i,j}$  and standard deviation  $\sigma_\ell$ , or Lévy truncated at a distance  $\ell_{\max,F}$  (generally smaller than  $\ell_{\max,L}$ ) (models C and D). Note that the correlation between the length and direction of the leader's and followers' movements can be essentially parameterized by  $\sigma_\theta$  and  $\sigma_\ell$  (in the Gaussian cases), and by  $\sigma_\theta$  and a couple of extra rules (see below) relating the  $C_j/R_j$ -ratio and the number of follower's steps (in the truncated Lévy cases). In particular, for a Gaussian  $p_F(\ell)$  function with  $\sigma_\theta = \sigma_\ell = 0$ , their moves become identical, i.e. the group as a whole displaces like a sole individual, right after the leader's first step (total correlation; absence of randomness in the spatial distribution of the followers around the leader:  $R_j = 0$  for  $j > 1$ ).

We now define some relevant quantities to describe the collective behavior of the leader and followers. By denoting the vector position of the follower  $i$  after  $j$  steps of the leader as  $\vec{r}_{i,j} = (x_{i,j}, y_{i,j})$ , the center of the mass of the set of  $N_F$  followers is calculated:

$$\vec{r}_{\text{CM},j} = \left( \frac{1}{N_F} \sum_{i=1}^{N_F} x_{i,j}, \frac{1}{N_F} \sum_{i=1}^{N_F} y_{i,j} \right). \quad (2)$$

The spatial distribution of the followers around the leader can be characterized by the two quantities mentioned above: (i) the mean radius  $R_j$  of the distribution of the followers about their center of mass, and (ii) the distance between their center of mass and the leader's position, or coefficient of separation,  $C_j$ . One can readily define  $R_j$  as

$$R_j = \frac{1}{N_f} \sum_{i=1}^{N_f} |\vec{r}_{i,j} - \vec{r}_{\text{CM},j}|, \quad (3)$$

which provides an indication of the group compactness. On the other hand, the coefficient of separation,

$$C_j = |\vec{r}_{L,j} - \vec{r}_{\text{CM},j}|, \quad (4)$$

gives a measure of how much the group behavior still follows the leader's trend after  $j$  steps. Indeed, strong collective behavior is characterized by relatively small values of both  $R_j$  and  $C_j$ . In addition, it is also interesting to define the average value of  $R_j$  over the whole walk:

$$\langle R \rangle = \frac{1}{N_L} \sum_{j=1}^{N_L} R_j, \quad (5)$$

where  $N_L$  is the number of leader's steps.

Although each follower takes its particular pathway in order to keep itself in the leader's vicinity, the average quantitative behavior can be inferred from the collective pattern as follows. First consider a type of 'mean follower' typical of the whole walk. Let  $\langle \ell_F \rangle$  and  $\langle \ell_L \rangle$  be the average step lengths of this follower and the leader. Also, denote by  $\langle \ell_{FL} \rangle$  and  $\langle \theta_{FL} \rangle$  the average distance and angle the follower should take to precisely reach the position of the leader after a single step. From figure 1(b) one can see that

$$\langle R \rangle^2 = \langle \ell_F \rangle^2 + \langle \ell_{FL} \rangle^2 - 2\langle \ell_F \rangle \langle \ell_{FL} \rangle \cos(\langle \theta_{FL} \rangle). \quad (6)$$

By writing

$$\langle \ell_{FL} \rangle^2 = \langle R \rangle^2 + \langle \ell_L \rangle^2, \quad (7)$$

we obtain

$$\langle R \rangle = \sqrt{(\langle \ell_F \rangle^2 + \langle \ell_L \rangle^2)^2 \left[ \frac{\sec(\langle \theta_{FL} \rangle)}{2\langle \ell_F \rangle} \right]^2 - \langle \ell_L \rangle^2}. \quad (8)$$

Thus, by analytically calculating  $\langle \ell_L \rangle$  and  $\langle \ell_F \rangle$  directly from the model definitions, and obtaining  $\langle \theta_{FL} \rangle$  from simulations, we can estimate the mean radius of the followers'

distribution around the leader, and compare the result with its numerically evaluated counterpart, equation (5).

### 2.1. Collective Brownian model with leader's Brownian behavior (model A)

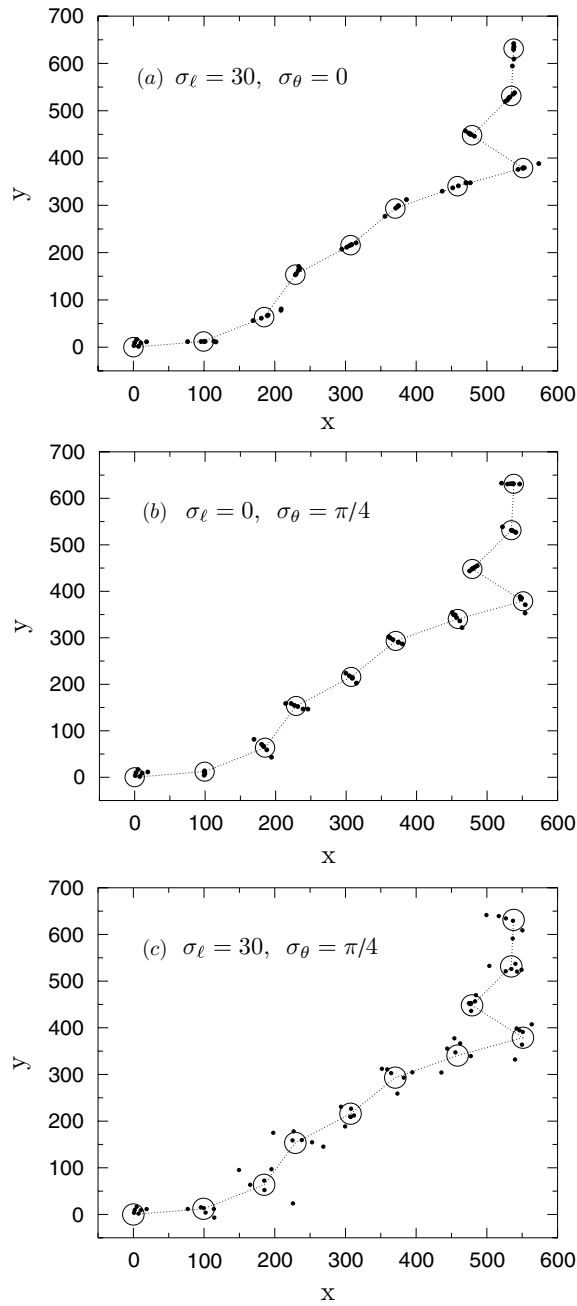
This model is characterized by Gaussian pdfs for both the angle and step length of the followers, respectively,  $\omega_F(\theta)$ , with mean  $\bar{\theta}_{i,j}$  and standard deviation  $\sigma_\theta$ , and  $p_F(\ell)$ , with mean  $\bar{\ell}_{i,j}$  and standard deviation  $\sigma_\ell$ . In this case,  $\bar{\ell}_{i,j}$  is chosen as the distance that should be taken by the follower  $i$  to precisely reach the leader's position after  $j$  steps. Further, the leader's behavior is determined by a uniformly random  $\omega_L(\theta)$  and a Gaussian  $p_L(\ell)$  as well, with mean  $\bar{\ell}_L$  and standard deviation  $\sigma_L$ . Actually, this is the only model in this work in which the leader's step length pdf is Gaussian; in models B, C and D below the leader takes a truncated Lévy  $p_L(\ell)$  function.

In figure 2 we illustrate a part of the path evolution of the leader (with  $\bar{\ell}_L = 100$  and  $\sigma_L = 0$ ) and six followers in three situations, namely (a)  $\sigma_\ell = 30$  and  $\sigma_\theta = 0$ ; (b)  $\sigma_\ell = 0$  and  $\sigma_\theta = \pi/4$ ; and (c)  $\sigma_\ell = 30$  and  $\sigma_\theta = \pi/4$ . We note in all cases a trend of the followers to remain grouped around the leader's position, even after a large number of steps, with the largest dispersion observed when both  $\sigma_\ell$  and  $\sigma_\theta$  are non-null. In fact, this result is related to the small standard deviation of the pdfs of both the (Gaussian) leader and followers, if compared with the large second moment of truncated Lévy distributions, which generally tends to make the followers disperse more easily with respect to the leader's path. Moreover, the relatively strong concentration of followers' step lengths about the mean  $\bar{\ell}_{i,j}$  in this Gaussian case causes each step of the leader to be generally accompanied by only a single step by each of the followers. This might not be the case in truncated Lévy models (see the discussion below on models C and D), in which either a difference in the indexes ( $\mu_L < \mu_F$ , where Brownian behavior of followers means  $\mu_F > 3$ ), or the choice of very distinct upper cutoff lengths ( $\ell_{\max,F} \ll \ell_{\max,L}$ ) imply the necessity of the followers to perform several steps in order to accompany one single jump of the leader.

Despite the strict maintenance of collective behavior in the present Brownian model, verified, for instance, through the observation of small changes in  $R_j$  and  $C_j$  with  $j$  (not shown), we note that the choice of Gaussian step length pdfs for both leader and followers does not generally result in efficient collective search patterns when target sites are scarce, similar to what occurs in searchers by sole individuals [1, 3]. In fact, truncated Lévy collective searches involving exponents  $\mu_L < 3$  and  $\mu_F < 3$  always lead to higher efficiency in this regime (see section 3). This is the reason why our focus in the next models is on Lévy rules of movement (step lengths), at least for the leader.

### 2.2. Collective mean-truncated-Lévy model with leader's truncated Lévy behavior (model B)

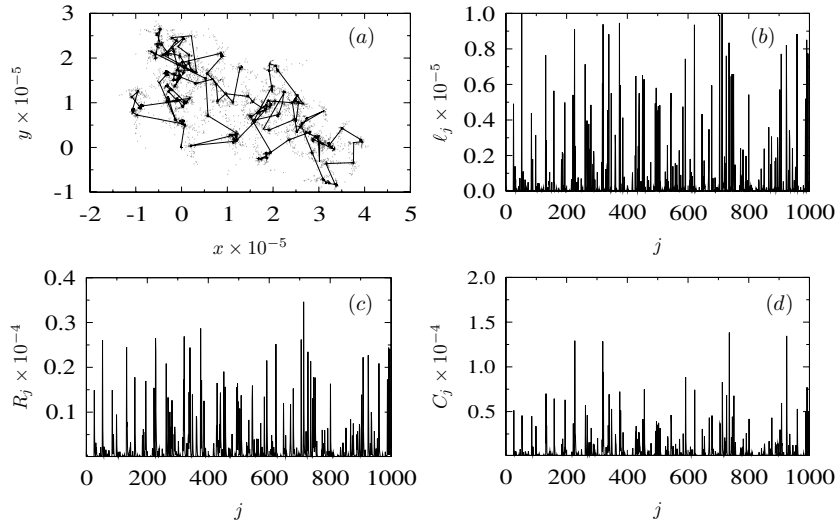
In this case, the leader's step length pdf is a truncated Lévy one, equation (1), and its angle distribution is uniformly random, as usual. On the other hand, the followers' step length and angle pdfs are Gaussian, with mean  $\bar{\ell}_{i,j}$  and standard deviation  $\sigma_\ell$  in principle defined just as in model A. However, since  $\bar{\ell}_{i,j}$  is now given by the distance  $\ell_{FL,ij}$  that should be taken by the follower  $i$  to reach the leader after a new single ( $j$ th) step, and considering that, in the present model, the distances  $\ell_{FL,ij}$  are (truncated) Lévy distributed (in contrast with the Gaussian distribution observed in model A), the followers' effective dynamics actually becomes faster than diffusive (Brownian), being also strongly driven by the leader's truncated Lévy evolution. In contrast, in model A the Brownian behavior of the leader induces genuine diffusive dynamics of the Gaussian followers.



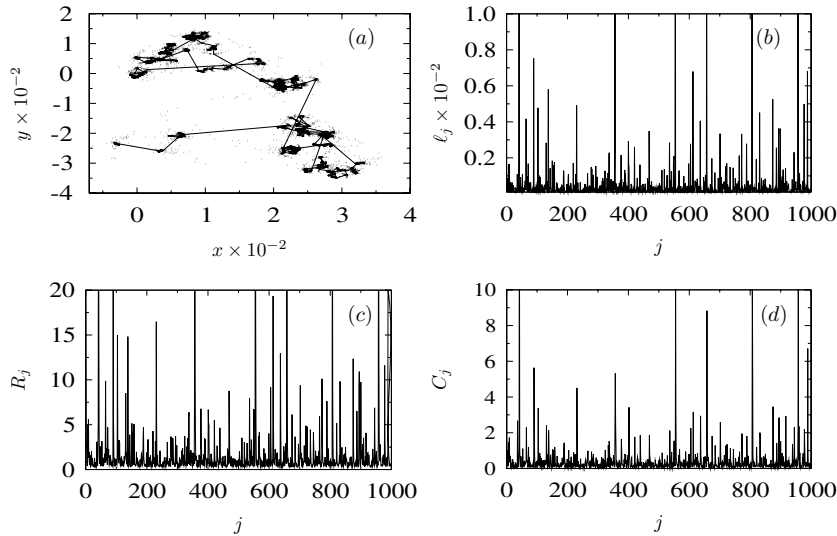
**Figure 2.** Collective Brownian model with leader’s Brownian behavior (model A). Illustration of typical paths with six followers, using  $\bar{\ell}_L = 100$ ,  $\sigma_L = 0$  and (a)  $\sigma_\ell = 30$  and  $\sigma_\theta = 0$ ; (b)  $\sigma_\ell = 0$  and  $\sigma_\theta = \pi/4$ ; (c)  $\sigma_\ell = 30$  and  $\sigma_\theta = \pi/4$ . The circumference around the leader is only a guide to the eyes.

Figures 3–5 show, respectively for  $\mu_L = 1.1, 2$  and  $3$ , the evolution as a function of  $j$  of the quantities  $\ell_j$  (of the leader),  $R_j$  and  $C_j$ , as well as the walk trajectory (leader and  $N_F = 32$





**Figure 3.** Collective mean-truncated-Lévy model with leader's truncated Lévy behavior (model B), using  $\mu_L = 1.1$ ,  $\ell_{\max,L} = 10^5$ ,  $r_v = 1$ ,  $\sigma_\ell = \ell_{FL,ij}/20$ ,  $\sigma_\theta = \pi/9$  and  $10^4$  leader's steps (only the first  $10^3$  are shown): (a) two-dimensional path; (b) sequence of leader's  $j$ th step lengths; (c) radius of the group  $R_j$ ; and (d) coefficient of separation  $C_j$ .



**Figure 4.** Same as in figure 3, for  $\mu_L = 2$ .

followers) in the two-dimensional plane. In all cases we have considered  $10^4$  leader's steps (only the first  $10^3$  are shown),  $\ell_{\max,L} = 10^5$ ,  $r_v = 1$ ,  $\sigma_\ell = \ell_{FL,ij}/20$  and  $\sigma_\theta = \pi/9$ .

Regarding the leader's dynamics, we observe in figures 3(a) and (b), 4(a) and (b) and 5(a) and (b) the typical pattern expected for (truncated) Lévy walks, with the nearly ballistic case for  $\mu_L = 1.1$  showing the presence of rare large jumps, although limited by  $\ell_{\max,L}$ , alternating between many short steps; as the value of  $\mu_L$  increases, the probability of such large jumps

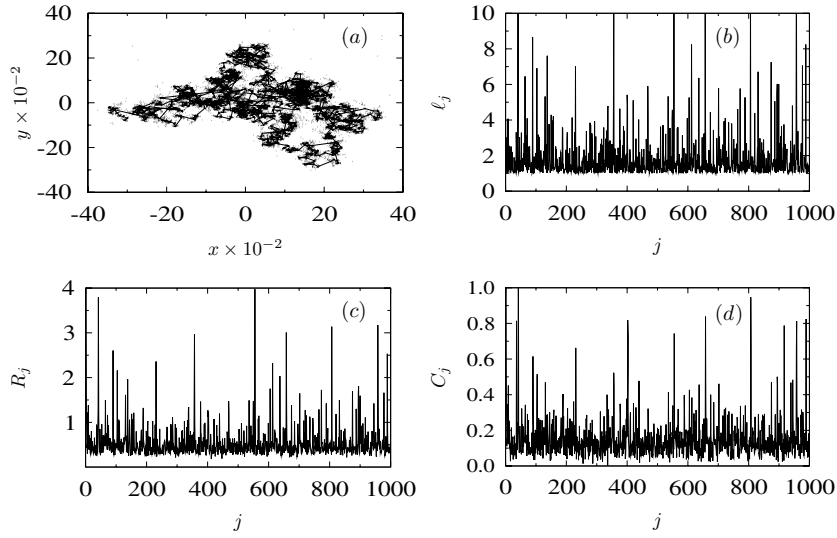


Figure 5. Same as in figure 3, for  $\mu_L = 3$ .

diminishes, and Brownian-like behavior emerges (see figures 5(a) and (b)). In all cases the dynamics of the leader is accompanied by the followers, as explained above, with each step of the leader corresponding to a single step by every follower.

The behavior of the mean radius of the group and coefficient of separation is shown in figures 3(c) and (d), 4(c) and (d) and 5(c) and (d). We first note that each large jump of the leader is essentially accompanied by a strong increase in both  $R_j$  and  $C_j$ . In this case the difficulty in grouping the followers around the leader gets higher (larger coefficient of separation), with simultaneous increasing of followers' spread about their center of mass (spatial distribution with larger average radius). This finding is also confirmed by the analysis of the histogram of  $R_j$  (not shown), which displays much larger standard deviation as  $\mu_L \rightarrow 1$  (compare, e.g., the typical ranges of  $R_j$  values:  $R_j \lesssim 4 \times 10^3$  for  $\mu_L = 1.1$  and  $R_j \lesssim 400$  for  $\mu_L = 3$ ). For  $\mu_L \rightarrow 3$  the radius of the group presents smaller deviation around the mean, also indicating that the influence of the leader on  $R_j$  is not so preponderant, in contrast with the case  $\mu_L \rightarrow 1$ .

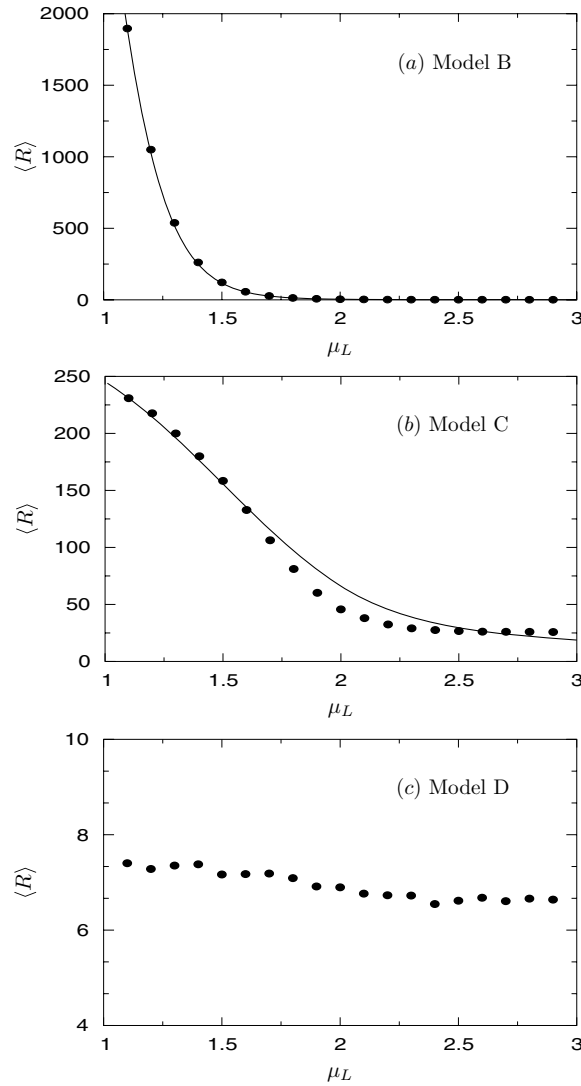
The average radius can be estimated in the present model by inserting into equation (8) the expression for the mean step length of the leader:

$$\langle \ell_L \rangle = \frac{(1 - \mu_L)(\ell_{\max,L}^{2-\mu_L} - r_v^{2-\mu_L})}{(2 - \mu_L)(\ell_{\max,L}^{1-\mu_L} - r_v^{1-\mu_L})}, \quad (9)$$

with the approximation  $\langle \ell_F \rangle \approx \langle \ell_L \rangle$  justified by the model rules. In addition, the numerical analysis of  $\langle \theta_{FL} \rangle$  provides

$$\langle \theta_{FL} \rangle = 2 + \frac{\pi \sigma_\theta}{2} - 2 e^{\alpha \sigma_\theta}, \quad (10)$$

with the fitting parameter  $\alpha = 0.316$  (the other numerical factors are adjusted so that  $\langle \ell_{FL} \rangle = 0$  for  $\sigma_\theta = 0$ —no angle dispersion around the straight line to the leader—and  $\langle \ell_{FL} \rangle$  nearly saturates close to 1.55 as  $\sigma_\theta \rightarrow \pi$ ). Figure 6(a) displays the nice agreement between the results of  $\langle R \rangle$  versus  $\mu_L$  from numerical simulations, with the use



**Figure 6.** Dependence of the mean radius of the group,  $\langle R \rangle$ , on the leader's Lévy exponent  $\mu_L$  (leader performs  $10^6$  steps; other parameters as in figures 3–5): (a) collective mean-truncated-Lévy model with leader's truncated Lévy behavior (model B); (b)–(c) collective truncated Lévy model with leader's truncated Lévy behavior ((b) continuous (model C) and (c) incremental (model D) versions), using  $\mu_F = 1.1$  and  $\ell_{\max,L} = 50$ . Solid lines represent analytical calculation of  $\langle R \rangle$  (see text).

of equation (5), and analytical, equations (8)–(10). (We have considered longer walks in figure 6, with the leader performing  $10^6$  steps; other parameters are as in figures 3–5.) As discussed, the mean radius increases considerably as larger jumps of the leader become more probable ( $\mu_L \rightarrow 1$ ), particularly for values  $\mu_L < 2$ ; in contrast, no significant variation in  $\langle R \rangle$  is observed for the nearly Gaussian dynamical regime of the leader ( $\mu_L \rightarrow 3$ ).

2.3. *Collective truncated Lévy model with leader's truncated Lévy behavior: continuous version (model C)*

We now consider both the leader and followers taking their step lengths from truncated Lévy distributions, with respective mean values given by equation (9) and

$$\langle \ell_F \rangle = \frac{(1 - \mu_F)(\ell_{\max,F}^{2-\mu_F} - r_v^{2-\mu_F})}{(2 - \mu_F)(\ell_{\max,F}^{1-\mu_F} - r_v^{1-\mu_F})}. \quad (11)$$

Note that, since  $\mu_L$  and  $\mu_F$  are arbitrarily chosen, the leader's and followers' dynamics are less correlated than in model B. We also choose the leader to possibly access much larger jumps:  $\ell_{\max,F} \ll \ell_{\max,L}$ . As above, the leader's and followers' angle pdfs are, respectively, uniformly random and Gaussian. In the continuous version of this model, at each new jump of the leader the dynamic rules are only applied to the followers upon total completion of the leader's step.

Due to the choice  $\ell_{\max,F} \ll \ell_{\max,L}$ , there is a trend for the followers to be left well behind the leader in the long term. To compensate this, and maintain the group's collective behavior, an extra rule should be imposed on the followers' dynamics: for a given step  $j$  of the leader, the followers must perform a number of steps until the coefficient of separation becomes smaller than the mean radius of the group,  $C_j < R_j$ . In other words, the followers must evolve until the leader's new position lies within the circumference of radius  $R_j$  and center at  $\vec{r}_{CM,j}$ . From figure 1(b), this mean number of steps can be estimated as

$$\langle N_s \rangle \sim \frac{\langle \ell_L \rangle}{\langle \ell_F \rangle \cos(\langle \theta_{FL} \rangle)}. \quad (12)$$

We further note that the large statistical fluctuations present in the truncated Lévy distribution may lead this condition to be fulfilled in a number of followers' steps much smaller than this estimate for  $\langle N_s \rangle$ , e.g. if some followers get too close to each other, whereas another subset of followers remains disperse. Thus, in order to group the followers in a compact way around the leader, we also require the condition  $C_j < R_j$  to be satisfied at a minimum choice of  $2\langle N_s \rangle/3$  steps. Only when both constraints are fulfilled can the leader take its next  $[(j + 1)\text{th}]$  step.

The mean radius  $\langle R \rangle$  as a function of  $\mu_L$  can be seen in figure 6(b), for  $\mu_F = 1.1$ ,  $\ell_{\max,L} = 10^5$ ,  $\ell_{\max,F} = 50$ ,  $r_v = 1$  and  $\sigma_\theta = \pi/4$ . Note that, since the angle deviation is not explicitly correlated with the step length distribution, equation (10) approximately applies to the present model as well. The analytical expression for  $\langle R \rangle$  is obtained by substituting equations (12) and  $\langle \ell_{FL} \rangle \sim \langle N_s \rangle \langle \ell_F \rangle$  into equation (6), thus resulting in

$$\langle R \rangle = \gamma \langle \ell_F \rangle \tan(\langle \theta_{FL} \rangle) + \beta, \quad (13)$$

where  $\gamma$  and  $\beta$  are numerical constants (dependent on the followers' specific behavior), eventually introduced in this mean approach in order to correct the results in the dispersionless limit  $\sigma_\theta = 0$ . Although still good, the fitting to the numerically evaluated  $\langle R \rangle$  from equation (5) is less perfect than that obtained using model B (figure 6(a)). This can be attributed to the more complex set of rules involved in model C.

By comparing figures 6(a) and (b), we observe that, whereas  $\langle R \rangle$  presents similar behavior in the Brownian limit  $\mu_L \rightarrow 3$  of both models B and C, the nearly ballistic regime with  $\mu_L \rightarrow 1$  displays very different magnitudes of the mean radius:  $\langle R \rangle \approx 2000$  for model B and  $\langle R \rangle \approx 240$  for model C. Indeed, the extra rules described above effectively act in the sense to keep relatively small values of model C's mean radius and coefficient of separation, even for low  $\mu_L$  and  $\mu_F$ . The results for  $R_j$  and  $C_j$  (not shown) also reflect this feature, with the presence of much smaller standard deviation for model C, as compared with that obtained from model B dynamics (for the same value of  $\mu_L$ ).

On the other hand, when results of simulations using different values of  $\mu_L$  in the present model are compared, a relevant feature becomes evident (similar to model B): Lévy leaders with  $\mu_L \lesssim 2$  tend considerably to disaggregate the group, as the increase in  $\langle R \rangle$  becomes pronounced in this range. This result has important consequences in the context of collective searches, discussed in section 3.

#### 2.4. Collective truncated Lévy model with leader's truncated Lévy behavior: incremental version (model D)

Finally we consider an incremental version of the preceding model, in which, if the length  $\ell_j$  of the leader's step  $j$  is larger than  $\langle \ell_F \rangle$ , then such a step is subdivided into  $\ell_j / \delta \ell_L$  parts, and, after the leader traverses each of these parts (and not only upon completion of the step), the dynamic rules are applied to the followers. Here we consider increments  $\delta \ell_L = a \langle \ell_F \rangle \cos(\langle \theta_{FL} \rangle)$  (see figure 1(b) and equation (12)), with the numerical choice  $a = 2/3$ .

As a consequence of this incremental rule, the followers' dispersion around the leader (quantified by  $R_j$  and  $C_j$ ) is greatly decreased, in comparison with the continuous version. This can be seen, e.g., by contrasting figures 6(b) and (c), obtained with the same parameters, in which the maximum value of  $\langle R \rangle$  in the incremental version is about 30 times smaller than that of the continuous version.

In addition, the reduced variation in the mean radius of model D in the range  $1 < \mu_L \leq 3$  indicates that, in the incremental version, the difficulty in maintaining the group compact around the leader is almost the same in the nearly ballistic and Gaussian regimes.

These findings have great relevance to the issue of compatibility between collective efficient searches and maintenance of group character, as discussed in the next section.

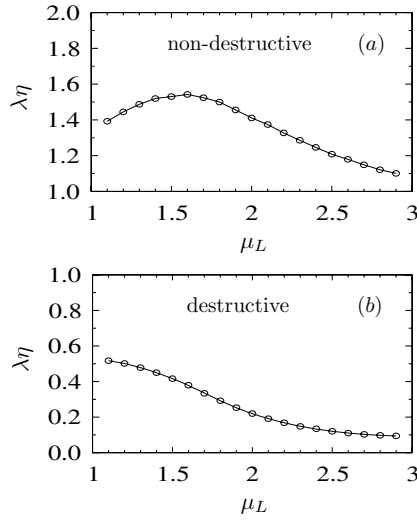
### 3. Efficiency study of collective search models

We now consider a group of foragers (leader and followers) looking for point target sites with uniformly random distribution in a two-dimensional search space. We are basically interested in studying the case in which the target density is low, compared with the mean free path of the foragers,  $\lambda$ . Indeed, very dense search spaces lead to the trivial result in which all foraging strategies are nearly as efficient, since one target site can always be found in the forager's close vicinity [3]. Further, as justified above, we consider the collective dynamics of the incremental version of the truncated Lévy model D.

In addition to the dynamics of model D, the searching and finding of sites by the leader and followers obey the rules below.

- (1) If there is a target site located within a 'direct vision' distance  $r_v$  of an element of the group (either the leader or a follower), then it moves on a straight line to the nearest target site.
- (2) If there is no target site within a distance  $r_v$  around the element, then it chooses a direction and a step length according to the rules of model D. It then incrementally moves to the new point, continually looking for a target within a radius  $r_v$  along its way. If it does not detect a target, it stops after completion of the step, and chooses a new direction and a new step length. Otherwise it proceeds to the target as in rule (1).

Note that, upon detection of a target along the search path, rule (2) contemplates the possibility of truncation of a step even before it is finished (not to be mistaken with the built-in upper truncation in the Lévy pdf). Rules (1) and (2) have been first applied to model the foraging dynamics by a sole individual [3]. Once found, a target can either be revisited any



**Figure 7.** Normalized efficiency,  $\lambda\eta$ , versus leader's Lévy exponent  $\mu_L$  for the incremental version of the collective truncated Lévy model with leader's truncated Lévy behavior (model D). Results are shown for (a) non-destructive and (b) destructive search, using  $N_F = 32$  followers,  $\mu_F = 1.1$ ,  $r_v = 1$ ,  $\ell_{\max,L} = 10^5$ ,  $\ell_{\max,F} = 4r_v N_F = 128$ ,  $\lambda = 5000$  and  $\sigma_\theta = \pi/4$ . Solid lines are guides for the eyes.

number of times (non-destructive foraging) or be destroyed (destructive foraging) [3]. In the latter case, another site is created at a random position, so as to keep the target density unaltered.

The efficiency function is defined [3] as the ratio between the quantity of targets found,  $N_{\text{found}}$ , and the total search path length (leader plus followers),  $L_{\text{tot}}$ , averaged over  $N_r$  simulation runs:

$$\eta = \frac{N_{\text{found}}}{L_{\text{tot}}}. \tag{14}$$

In figure 7 we plot the normalized efficiency versus  $\mu_L$ , for (a) non-destructive and (b) destructive cases. We have considered  $N_F = 32$  followers, with  $\mu_F = 1.1$ ,  $\ell_{\max,L} = 10^5$ ,  $\lambda = 5000$ ,  $r_v = 1$  and  $\sigma_\theta = \pi/4$ . Each simulation run ended upon the finding of  $N_{\text{found}} = 10^4$  targets, with average taken over (a)  $N_r = 10^4$  and (b)  $N_r = 10^3$  realizations. In addition, it is also interesting to study the behavior of the search efficiency with the number of followers,  $N_F$ . In realistic contexts, it is not expected for the followers to increase considerably their density with a fixed  $\langle R \rangle$  as  $N_F$  grows; otherwise the global search efficiency would tend to decrease, due to the limitation in the number and diversity of targets found (more individuals searching in the same area and sharing the intake). Some kind of subtle ‘repulsion force’ between the followers must then be present to circumvent this potential difficulty. In our model approach, this can be accomplished by setting, e.g., the number of followers to be proportional to the individual upper cutoff length in the truncated Lévy distribution:  $\ell_{\max,F} = 4r_v N_F$ . As a result, for a larger  $N_F$  the mean distance between followers grows, resulting in a more extensive searched area around the leader.

It is interesting to note in figure 7 that, for  $N_F = 32$ , the qualitative behavior of  $\eta$  resembles that of sole foraging [5], with the achievement of the maximum efficiency dislocating from

$\mu_L \lesssim 2$  (non-destructive search) to  $\mu_L \rightarrow 1$  (destructive). In any case, Brownian searches ( $\mu_L > 3$ ) are shown to be rather inefficient.

Nevertheless, one important distinction between sole and collective foraging arises due to the inherent necessity to keep the group together in the latter. Indeed, we have noticed in models A, B and C above the general increasing trend of the mean radius of the group for  $\mu_L \lesssim 2$ , which tends to destabilize the collective behavior in the long term, and to give rise to a set of weakly-interacting individuals considerably distant from each other. This constraint might potentially represent an actual problem whenever searches with increasing ballistic feature of the leader become the optimal choice from the individuals' point of view. However, this drawback can be essentially eliminated by applying, e.g., an incremental dynamic strategy as in model D. In this case, the optimal collective efficiency search strategy is achieved for a value  $\mu_{\text{opt},L}$  related to a mean radius of the group essentially of the same size as the ones of (much less efficient) Gaussian strategies: from figure 6(c),  $\langle R \rangle \approx 7.1$  for  $\mu_{\text{opt},L} = 1.6$ , whereas  $\langle R \rangle \approx 6.7$  for  $\mu_L = 2.9$ .

In the discussion above, as the number of followers grows, so does the total effective area swept by the group, implying a type of rescaling of the individual radius of vision,  $r_v$ . In particular, in the limit in which the mean distance between followers approaches the average separation between targets, there can possibly exist a target site in the vicinity of some element (leader or follower). In this case, the rate of truncation of steps by the finding of a target increases, and the dependence of the efficiency  $\eta$  on  $\mu_L$  becomes weaker (as found, e.g., in simulations with four times more followers). As a consequence, we notice that as the number of followers grows, the optimal exponent tends to shift towards  $\mu_L \rightarrow 1$ .

Taking these considerations into account, we conclude that a balance must be achieved between the trend to keep the group together and behaving in a collective fashion, and the global efficiency of the search, which, depending on specific details of the foraging process (e.g. regeneration of target sites, energy dissipative function, etc), can be maximized for a value of  $\mu_L$  in the interval  $1 < \mu_L \lesssim 2$ . In this sense, extra constraints in the way the group elements perform their search paths can actually keep the radius of the group and coefficient of separation suitable to avoid followers' dispersion, even in the low- $\mu_L$  regime. Lévy strategies and collective behavior can thus be compatible in the context of optimum random searches.

#### 4. Conclusions

We have presented in this work four models of collective behavior of a leader and followers, comprising both Gaussian and truncated Lévy dynamics.

From the unique viewpoint of keeping the group aggregated together, Lévy collective behavior has shown to be more dispersive than the Gaussian one. Therefore, regarding collective searches in environments with low densities of target sites, situations may appear in which a Lévy dynamics of collective movement might conflict with the optimal search strategy, with tendency to group dispersion and loss of collective character in the long term.

In spite of this, we have shown in a simplified context that extra rules imposed on the dynamics of the group elements can actually make both efficient Lévy strategies and collective search compatible. The identification of such rules in realistic (rather complex) scenarios is an important line of investigation in random search theory. In this sense, further studies are still needed in order to clarify this issue.

Finally, we mention a possible extension of the present work, related to the choice of leadership during the random search. In a purely rational association between different elements of a group, the leader should naturally be the most skillful member. This seems to be the case [14] when social ties are weak or absent. On the other hand, many animal

species [23] define a hierarchical structure reliant on dominance and affiliation [24]. It might eventually happen that such ‘elected’ leader is not the most successful individual for foraging. A model considering the performance of each element during the search, taking into account the advantages to switch to a new (contextual) more efficient leader and also the costs of breaking the already established bounds, could be an interesting way to study the balance between social relations and optimization goals within a group, including the interesting possibility of group fission [24].

## Acknowledgments

We thank CNPq, CAPES, Finep (CT-Infra, project MCC-VFPR), FAPEAL, FACEPE and Fundação Araucária (Brazilian agencies) for financial support.

## References

- [1] For a recent review, see Viswanathan G M, Raposo E P and da Luz M G E 2008 *Phys. Life Rev.* **5** 133
- [2] Shlesinger M F and Klafter J 1986 *On Growth and Form* ed H E Stanley and N Ostrowsky (Dordrecht: Nijhoff) p 279  
Shlesinger M F, Zaslavsky G M and Klafter J 1993 *Nature* **363** 31  
Shlesinger M F, Zaslavsky G and Frisch U (eds) 1995 *Lévy Flights and Related Topics in Physics* (Berlin: Springer)
- [3] Viswanathan G M, Buldyrev S, Havlin S, da Luz M G E, Raposo E P and Stanley H E 1999 *Nature* **401** 911
- [4] Raposo E P, Buldyrev S V, da Luz M G E, Santos M C, Stanley H E and Viswanathan G M 2003 *Phys. Rev. Lett.* **91** 240601
- [5] Santos M C, Raposo E P, Viswanathan G M and da Luz M G E 2004 *Europhys. Lett.* **67** 734
- [6] da Luz M G E, Buldyrev S V, Havlin S, Raposo E P, Stanley H E and Viswanathan G M 2001 *Physica A* **89** 295
- [7] Giraldeau L-A and Caraco T 2000 *Social Foraging Theory* (Princeton, NJ: Princeton University Press)
- [8] Ranta S, Rita H and Peuhkuri 1995 *Behav. Ecol.* **6** 1  
Reynolds A M 2006 *Phys. Lett. A* **354** 384
- [9] Zhang N and Wunsch D C II 2006 *Advances in Neural Networks: ISNN 2006—Part II* ed Z Wang, Z Yi, J M Zurada, B L Lu and Y Hujun (Berlin: Springer) p 1140
- [10] Bulka B, Gaston M and Desjardins M 2007 *Auton. Agents Multi-Agent Syst.* **15** 29
- [11] Detrain C and Deneubourg J-L 2006 *Phys. Life Rev.* **3** 162
- [12] Cui X, Patton R M, Treadwell J and Potok T E 2008 *Stud. Comput. Intell.* **129** 211
- [13] Faustino C L, Silva L R da, Luz M G E da, Raposo E P and Viswanathan G M 2007 *Europhys. Lett.* **77** 30002
- [14] Quenette P Y, Ferron J and Sirois S 1997 *Behav. Process.* **41** 29
- [15] Seabloom E W and Reichman O J 2001 *Am. Nat.* **157** 76
- [16] Ramos-Fernandez G, Mateos J L, Miramontes O, Cocho G, Larralde H and Ayala Orozco B 2004 *Behav. Ecol. Sociobiol.* **55** 223
- [17] Boyer D, Miramontes O, Ramos-Fernandez G, Mateos J L and Cocho G 2004 *Physica A* **342** 329
- [18] Ramos-Fernandez G, Boyer D and Gomez V P 2006 *Behav. Ecol. Sociobiol.* **60** 536
- [19] Boyer D, Ramos-Fernandez G, Miramontes O, Mateos J L, Cocho G, Larralde H, Ramos H and Rojas F 2006 *Proc. R. Soc. Lond. B* **273** 1743
- [20] Seth A K 2002 *Adapt. Behav.* **9** 067
- [21] Ruxton G D and Glasbey C A 1995 *Math. Med. Biol.* **12** 71
- [22] Sims D W *et al* 2008 *Nature* **451** 1098  
See also Edwards A M *et al* 2007 *Nature* **449** 1044
- [23] Conradt L and Roper T J 2003 *Nature* **421** 155  
Schaller G 1963 *The Mountain Gorilla: Ecology and Behavior* (Chicago: University of Chicago Press)  
Prins H H T 1996 *Ecology and Behaviour of the African Buffalo* (Boca Raton, FL: Chapman & Hall)
- [24] King A J, Douglas C M S, Huchard E, Isaac N J B and Cowlshaw G 2008 *Curr. Biol.* **18** 1833
- [25] Shi H, Wang L and Chu T 2006 *Physica D* **213** 51
- [26] Viswanathan G M, Raposo E P, Bartumeus F, Catalan J and da Luz M G E 2005 *Phys. Rev. E* **72** 011111
- [27] Mantegna R N and Stanley H E 1994 *Phys. Rev. Lett.* **73** 2946
- [28] Bartumeus F, Catalan J, Viswanathan G M, Raposo E P and da Luz M G E 2008 *J. Theor. Biol.* **43** 252
- [29] Buldyrev S V, Goldberger A L, Havlin S, Peng C-K, Simmons M and Stanley H E 1993 *Phys. Rev. E* **47** 4514